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Short Communication

Dynamic model of the grinding process

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Abstract

Grinding is one of the most versatile methods of removing material from machine parts to provide precise geometry. Dynamic analysis of the grinding process is necessary to reduce the surface waviness and roughness induced by vibrations, and to offer a machining accuracy in the order of nanometers.

This research is to investigate the dynamic function of the grinding process. A new approach to determination of cutting factors in dynamic grinding is proposed. Attention is paid to the mechanisms of dynamic grinding from the kinematics viewpoint. A non-linear dynamic model is developed to investigate the dynamic characteristics of the grinding process. The model demonstrates that different vibration frequencies result in qualitatively different behavior of the grinding machine. The relationship between grinding force variations and vibration frequency is revealed. The formulas to calculate cutting force variations are given. A comparison of the theoretical transfer function of dynamic grinding and experimental one shows good matching.

As a result, the paper significantly expands the opportunities of vibration control of grinding machines.

1. Introduction

Grinding is one of the most versatile methods of removing material from machine parts to provide precise geometry. There is a vast body of theoretical and empirical information in the literature to assist process planners to select the cutting parameters for grinding operations. However, little attention has been given to development of knowledge-based systems that allow for efficient selection of cutting parameters based directly on the work and cutting tool material properties.

To obtain components with close tolerances it is necessary to perform dynamic analysis of cutting machines taking into account the dynamic transfer function of the cutting process [1–11].

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Dynamic analysis of the grinding process is also necessary to reduce the surface waviness and roughness induced by vibrations, and to offer a machining accuracy in the order of nanometers [5,6].

In our research attention is paid to the mechanisms of dynamic grinding from the kinematics viewpoint. The process of grinding is based on the relative movements that are performed by every cutting grain of the grinding wheel and the grounded component. One of the methods that can be used to obtain the dynamic transfer function of the grinding process is to analyze the process of transforming relative vibrational movements of the grinding wheel and the component into instant characteristics of cutting which determine plastic deformation of grounded material [6]. Relative vibrations cause changes in the prescribed kinematics of the process and, consequently, changes in the two most important parameters of grinding: the number of grains concurrently performing cutting at the length of the contact of the grinding wheel (i_z) and the component and chip thickness (a_{zm}).

When analyzing the above-mentioned process the following assumptions can be made: (a) before grinding the grinding wheel and the grounded component have ideal geometry; (b) the curvedness of the trajectory of the relative movement of the center of the grinding wheel and the grounded component is less than the curvedness of the grinding wheel; (c) the period of the relative vibrations of the instrument and the component is at least two times higher than the time necessary to form the working part of the trajectory of the grains of the grinding wheel.

2. Dynamic model

The theoretical thickness (a_{zm}) of the undeformed chips which are obtained under the prescribed (ideal) kinematics conditions, can be calculated using the following formulas [6]:

$$a_{zm} = 1.41dk_\delta^{0.5}(v/V)^{0.5}(t/D)^{0.25} \quad \text{if } 10^{-6} \leqslant H \leqslant 10^{-5}, \quad n = 1, \tag{1}$$

$$a_{zm} = 1.32dk_\delta^{0.4}(v/V)^{0.4}(t/D)^{0.2} \quad \text{if } H = 0, \quad n = 1.5, \tag{2}$$

where d is the size of the grain, n the coefficient depending on the deepness of the cutting profile, H the difference in the heights of two adjacent grains, k_δ the coefficient depending on the grinding wheel marking and the conditions of dressing, D the diameter of the grinding wheel, v the speed of the relative movement of the grinding wheel and the grounded component (at the absence of vibrations $v = v_0$), V the cutting speed and t the depth of cut (at the absence of vibrations $t = t_0$) (see Fig. 1).

To ensure that parameters (1) and (2) have their physical sense at the presence of relative vibrational movements of the grinding wheel and the grounded component, it is necessary to determine parameters t and v in the system of co-ordinates in which equations of the trajectories of the grains are the same as those in ideal kinematics. The equations of the two adjacent grains G1 and G2 can be described as follows [6]:

$$Z_{G1} = r_o\varphi_A + \frac{D}{2}\sin\varphi_A,$$
$$Y_{G1} = r_o + \frac{D}{2}\cos\varphi_A,$$

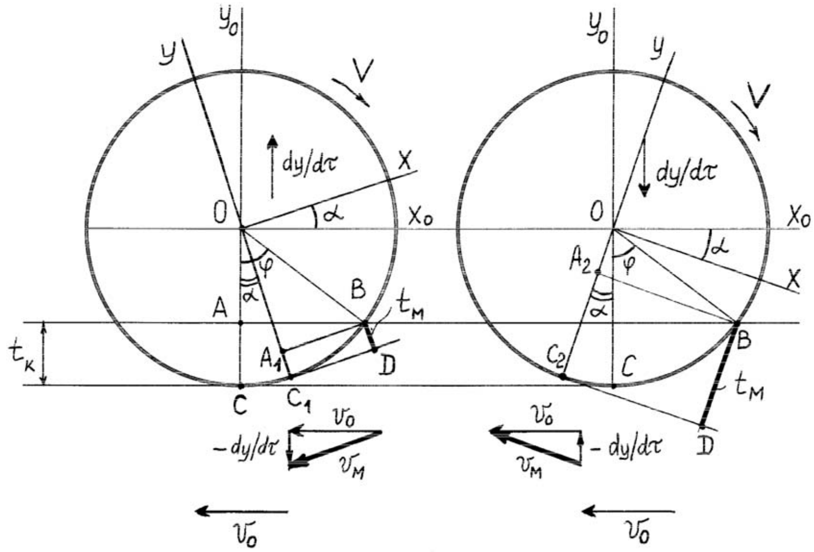

Fig. 1. Schemes to determine instant characteristics in dynamic grinding: (a) $dy/d\tau > 0$; (b) $dy/d\tau < 0$.

$$Z_{G2} = r_o(\varphi_B - v) + \left(\frac{D}{2} \mp H\right)\sin\varphi_B,$$

$$Y_{G2} = r_o + \left(\frac{D}{2} \mp H\right)\cos\varphi_B, \tag{3}$$

where $r_o = v/\omega_K$, ω_K is the speed of rotation of the grinding wheel, φ_A the changing angle of the contact $0 \leqslant \varphi_A \leqslant \varphi_{\max}$, $\varphi_{\max} = (4t/D)^{0.5}$. Eq. (3) determine relative positions of the cutting parts of the trajectories of the two adjacent grains and, consequently, the chip thickness under dynamic grinding if we use the following equations:

$$v = v_M = \sqrt{v_o^2 + (dy/d\tau)^2},$$

$$t = t_M, \tag{4}$$

where $dy/d\tau$ is the speed of the relative movement of the grinding wheel and the component along the axis OY_O. So at the presence of relative vibrational movements of the grinding wheel and the grounded component cutting conditions can be determined as follows (see Fig. 1). Relative instant speed of cutting v_M (that is the speed of the relative movement of the grinding wheel and the component) is the composition of the vector of the prescribed speed of cutting v_O and the vector which is equal and opposite to the vector of the speed of their relative vibrational movement along axis $OY_O(dy/d\tau)$. Instant depth of cut t_M is the projection of the length of the contact onto the axis that is perpendicular to the vector of the relative instant speed of cutting v_M .

When using parameters $v = v_M$ and $t = t_M$ in Eqs. (1) and (2), parameter a_{zm} determines the distance between the trajectories of the two adjacent grains, that is the thickness of the chip cut by one grain. This makes it possible to use functions for dynamic cutting forces that include parameters a_{zm} and i_z.

The instant depth of cut can be determined as follows. If $\mathrm{d}y/\mathrm{d}\tau > 0$ (Fig. 1) then from ΔOAB the following parameters can be found:

$$\begin{aligned} &OB = OC = D/2,\\ &AB = \sqrt{OB^2 - OA^2},\\ &OA = D/2 - t_K,\\ &AB = OB\sin\varphi. \end{aligned}$$

Consequently,

$$\varphi = \arcsin\frac{2\sqrt{(D - t_K)t_K}}{D}. \tag{5}$$

Using ΔOA_1B the following parameters can be found:

$$\begin{aligned} &A_1B = \sqrt{OB^2 - OA_1^2},\\ &OA_1 = D/2 - t_M,\\ &A_1B = OB\sin(\varphi - \alpha), \end{aligned}$$

$$\varphi - \alpha = \arcsin\frac{2\sqrt{(D - t_M)t_M}}{D}. \tag{6}$$

The angle α can be found from the formula

$$\alpha = \arcsin\frac{\mathrm{d}y/\mathrm{d}\tau}{\sqrt{v_0^2 + (\mathrm{d}y/\mathrm{d}\tau)^2}}. \tag{7}$$

Taking into account that in grinding $D \gg t_M$; $D \gg t_K$; $\sin\varphi \cong \varphi$; $\sin\alpha \cong \alpha$ and using Eqs. (5)–(7) we have the following formula to determine the instant depth of cut:

$$t_M = t_K - \frac{\mathrm{d}y/\mathrm{d}\tau}{\sqrt{v_0^2 + (\mathrm{d}y/\mathrm{d}\tau)^2}}\sqrt{Dt_K} + \frac{D}{4}\frac{(\mathrm{d}y/\mathrm{d}\tau)^2}{v_0^2 + (\mathrm{d}y/\mathrm{d}\tau)^2}, \tag{8}$$

where

$$t_K = t_0 - y \tag{9}$$

and y is the relative movement of the grinding wheel and the grounded component (Fig. 1). The same calculations can be made for $\mathrm{d}y/\mathrm{d}\tau < 0$. The instant speed of cut can be found using Eq. (4). Taking into account that $v_0 \gg |\mathrm{d}y/\mathrm{d}\tau|$ it can be assumed that $v_M \cong v_0$. Consequently, the changes in the cutting force are basically caused by changing the depth of cut.

3. Fourier analysis

Analysis of Eq. (8) shows that the process has non-linear character. Harmonic components caused by DTF can be determined using Fourier analysis

$$F(\tau) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \sin n\omega\tau + b_n \cos n\omega\tau). \tag{10}$$

Assuming that

$$\Delta t_K = A \sin \omega\tau \tag{11}$$

and taking into account that $\Delta t_K = -y$ we can obtain the following formula to calculate the dynamic component of the depth of cut:

$$\Delta t_M \cong A \sin \omega\tau + \frac{Aw\sqrt{Dt_0}}{v_0} \cos \omega\tau + \frac{A^2\omega}{2\omega_0}\sqrt{\frac{D}{t_0}} \sin \omega\tau \cos \omega\tau + \frac{DA^2\omega^2}{4v_0^2} \cos^2 \omega\tau. \tag{12}$$

The Fourier coefficients can be calculated as follows:

$$a_0 = \frac{D}{4}\left(\frac{A\omega}{v_0}\right)^2, \quad a_1 = A, \quad a_2 = \frac{A^2\omega}{4v_0}\sqrt{\frac{D}{t_0}},$$
$$b_1 = \frac{\omega A\sqrt{Dt_0}}{v_0}, \quad b_2 = \frac{DA^2\omega^2}{8v_0^2}. \tag{13}$$

The first harmonic component that can be used to determine the dynamic coefficient of the grinding process is equal to

$$|\Delta t_M| = \sqrt{a_1^2 + b_1^2}. \tag{14}$$

The angle can be calculated using (13):

$$\Phi = \text{arctg}\frac{b_1}{a_1}. \tag{15}$$

Taking into account Eqs. (13)–(15) we can obtain the following final formula to determine the dynamic component of the depth of cut in dynamic grinding:

$$\Delta t_M = A\sqrt{1 + Dt_0\left(\frac{\omega}{v_0}\right)^2} \sin\left(\omega\tau + \text{arctg}\frac{\omega\sqrt{Dt_0}}{v_0}\right). \tag{16}$$

4. Discussion

Analysis of Eq. (16) shows that in dynamic grinding the changes in the cutting force is ahead of the relative vibration of the grinding wheel and the grounded component.

Generally, in grinding the dynamic distance between two adjacent grains is much higher than changes in the length of the contact between the grinding wheel and the component that are caused by their relative vibration. Consequently, we can assume that in dynamic grinding the number of grains concurrently performing cutting does not change as compared to grinding with ideal kinematics. So it can be assumed that the changes in the cutting force are caused by the changes in the chip thickness. However, analysis of Eqs. (1) and (2) shows that the influence of the depth of cut on the grinding force is not so essential (raising to a power of 0.2–0.25). As a result, disregard to essential changes in the depth of cutting, the dynamic component of the grinding force does not significantly change with changes in the frequency of vibration.

Taking into account that the forces that effect a grain are determined by the chip thickness, the amplitude of the grinding force can be calculated using the following formula:

$$\Delta P = \frac{\Delta a_{zm}}{\Delta a_{zm}^0} K_p A, \tag{17}$$

where K_p is the static stiffness of grinding; Δa_{zm}^0 is an increase in the chip thickness when the depth of cut changes according to $\Delta t = A$; Δa_{zm} is an increase in the chip thickness under vibration

$$\Delta t = A\sqrt{1 + Dt_0\left(\frac{\omega}{v_0}\right)^2}.$$

Consequently, the dynamic coefficients can be determined using the following formula:

$$K_D = \frac{\Delta P}{\Delta t_K} = K_P\left(1 + \left(\frac{\omega}{v_0}\right)^2 Dt_0\right)^{\beta},$$

$$\beta = \begin{cases} 0.125 & (n = 1), \\ 0.1 & (n = 1.5). \end{cases} \tag{18}$$

Finally, amplitude–frequency characteristic of the grinding process can be found from Eq. (18), phase–frequency characteristic is determined by (15)

$$\Delta\varphi = \operatorname{arctg}\frac{\omega\sqrt{Dt_0}}{v_0}. \tag{19}$$

5. Example and experimental results

For example, determine the dynamic characteristic of the grinding process for following grinding conditions: grinding process—surface grinding, wheel shape—straight; diameter of the grinding wheel—$D = 250$ mm, abrasive material—synthetic aluminum oxide, grain size—40 (0.40–0.50 mm), bond—vitrified, grinding wheel grade—CM1, cutting speed—$V = 30$–35 m/s, the speed $v_0 = 10$ m/min, cross-feed $S_x = 8.5$ mm, the depth of cut $t_0 = 0.005$–0.03 mm, the amplitude of vibration $A = 0.001$–0.002 mm, and the frequency of vibration $f = 0$–120 Hz.

The static stiffness of grinding for the above conditions is equal to 7 H/mkm [5,6]. Amplitude–frequency characteristic of the grinding process, calculated according to Eq. (18), are shown in Fig. 2. The phase–frequency characteristics, calculated according to Eq. (19), are shown in Fig. 3.

As it can be seen from the graphs, the comparison of the theoretical transfer function of the grinding process and the experimental ones shows good matching in the considered range of frequency of vibration.

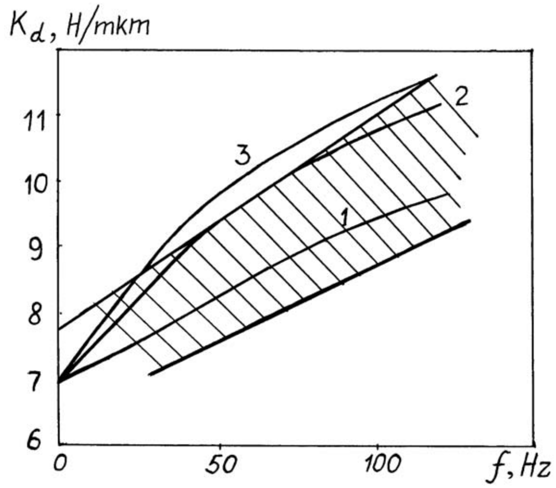

Fig. 2. Amplitude–frequency characteristics of the grinding process. Theoretical results: (1) $t = 0.005$ mm; (2) $t = 0.02$ mm; (3) $t = 0.03$ mm. Experimental results [5]: ▧.

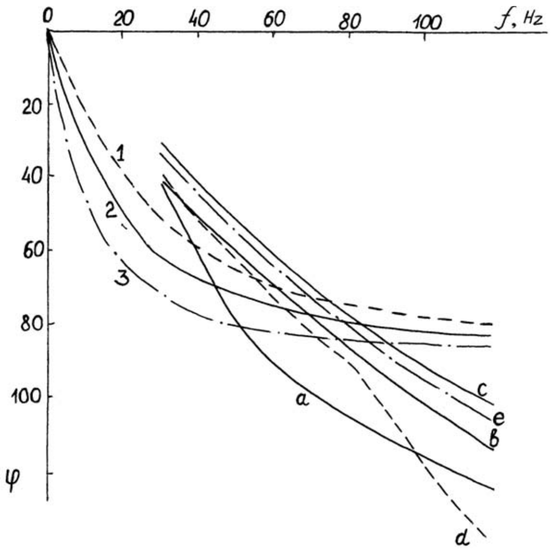

Fig. 3. Phase–frequency characteristics of the grinding process. Theoretical results: (1) $t = 0.005$ mm; (2) $t = 0.01$ mm; (3) $t = 0.03$ mm. Experimental results [5]: (a) Y8A (HRC 54–56); (b) 40X (HRC 45–48); (c) 45 (HB 230–270); (d) IIIX15 (HRC 58–62), (e) C 21–40 (HB 180–220).

6. Conclusions

This paper has developed a non-linear model for the analysis of dynamic characteristics of grinding processes. The obtained analytical model shows that different vibration frequencies result in qualitatively different behaviour of grinding machines. The analytical results of the transfer function of dynamic grinding are compared to those obtained experimental and it can concluded that the developed model is valid in the considered range of frequency of vibration. Future research effort will made on the use of this gained knowledge to the vibration suspensions of grinding machines.

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